SOME ASPECTS OF THE INTERACTIONS OF FAST CHARGED PARTICLES WITH MATTER

by

ARI BRYNJOLFSSON

The Niels Bohr Institute

Available at HARVARD COOPERATIVE SOCIETY, Harvard Square, Cambridge, Massachusetts 02138, U.S.A.
Price: $3.60 per copy

Denne afhandling er af det naturvidenskabelige fakutetsråd ved Københavns universitet antaget til offentlig at forsvares for den filosofiske doktorgrad.

København, den 19. marts 1973.

Morten Lange

h.a.dec.

SOME ASPECTS OF THE INTERACTIONS OF FAST CHARGED PARTICLES WITH MATTER

by

ARI BRYNJOLFSSON

The Niels Bohr Institute

CONTENTS

CONTENTS (cont'd)

INTRODUCTION

The primary interactions of incident charged particles with matter have played a most important role in the development of our knowledge of the microscopic world and a great many authors have contributed to the evolution of the theory. Most outstanding of these contributions are: a) Niels Bohr's classical treatment for nonrelativistic particles [Bo13a] (published shortly before his article "On the Constitution of Atoms and Molecules" [Bo13b]), and his extension of these calculations to relativistic particles [Bo15]; b) Hans Bethe's quantum mechanical treatment of the interaction of nonrelativistic incident particles [Be30]; c) Christian Møller's relativistic extension of Bethe's calculations [Mø32a] and [Mø32b]; d) Enrico Fermi's classical treatment of the interaction of incident charged particles with ponderable matter [Fe39] and [Fe40]. Other important theoretical contributions are Bloch's adaptation of the Thomas-Fermi statistical model of the atom for his treatment of the dynamics of interactions [Bl33a] and [Bl33b], Bhabha's scattering of positrons by electrons [Bh36] and Frank and Tamm's theory explaining Čerenkov radiation [FT37] and [Ta39]. The subject has been surveyed among others by Bethe [Be33], Livingston and Bethe [LB37], N. Bohr [Bo48a], A. Bohr [Bo48b], Bethe and Ashkin [BA53], Evans [Ev55], Massey [Ma56], Birkhoff [Bi48], Fano [Fa63], Fano and Cooper [FC68] and Moiseiwitsch and Smith [MS68].

The quantum theory of fast particles with matter as developed by Bethe and Møller rests on the assumption that the incident particle's speed is much greater then the "speed" of the atomic electrons in their normal bound state. In the case of heavy particles — protons, α-particles and fission fragments — this assumption is not fulfilled. The subject has been dealt with among others by Bohr [Bo48a] by Lindhard *et al.* [LSS63], and reviewed by Northcliffe [No63].

In Chapter I we will consider for incident electrons the quantum mechanical prediction of ionization and excitation cross sections, and the energy per ion formation, in particular interactions with individual hydrogen atoms for which the theoretical calculations are most accurate.

In the formulation of the phenomena in ponderable matter by Frank and Tamm and by Fermi the matter is described as a continuum, while Aage Bohr [Bo48b] considered the same phenomena from the atomic point of view. In Chapter III we will elaborate and extend A. Bohr's viewpoints. First, however, in Chapter II, we will elaborate and present a new formulation of N. Bohr's classical treatment and relate this formulation closely to the quantum mechanical treatment. Using Bohr's semiclassical quantum rules for angular momentum we succeed in deducing from Bohr's classical stopping power equation some of the corresponding quantum mechanical results. In Chapter III we extend the formulation from Chapter II of the interaction phenomena with individual oscillators to a theory for charged particle interactions with ponderable matter consisting of a single type of oscillators, and in Chapter IV we extend the formulation further to include ponderable matter with a variety of oscillator types.

In ponderable matter this formulation results in significant corrections to the stopping power equations in the current literature. Quantum mechanical considerations result in a relativistic correction Δ_4 which depends on the energy of the incident particle. This correction can explain the inconsistency between the stopping power measurements of 10–30 MeV protons and 100–600 MeV protons when these are compared with previous theoretical formula. It is shown further that coupling between oscillators of different frequencies results in a correction Δ_3 which is significant when comparing the theory with exact measurements. In Section 10 of Chapter IV the present formulation is applied to the stopping of charged particles in helium and aluminum. The agreement between the purely theoretical and the best experimental results is shown to be very good.

It has been most rewarding, in this thesis, to be able to show the very close relation between the different theories, for example between Bohr's old stopping power equations, and Bethe's and Møller's quantum mechanical results, and finally to show how the formulation used in sparse matter could be applied with small modifications to ponderable matter to obtain through microscopic analysis of the interactions phenomena the equations valid for these including the Čerenkov radiation. Although we limit the description to electrons in Chapter I for reasons of clarity, we have given in Chapter II, III and IV a very general description of the particles, indicating the wide applicability of the results.

CHAPTER I

QUANTUM MECHANICAL RESULTS OF THE PRIMARY INTERACTIONS OF FAST ELECTRONS WITH INDIVIDUAL ATOMS

1. INTRODUCTION

The theory for treating the collision phenomena evolved simultaneously with a quantum mechanics. Born's idea of treating quantum-mechanically the collision phenomena as perturbation ([B26a] and [B26b]) inspired Bethe to undertake a thorough treatment of the collisions of incident fast nonrelativistic charged particles with individual atoms using Born's first approximation [Be30]. Using Dirac's quantum mechanical formulation and Born's first approximation, Møller extended Bethe's treatment to relativistic incident particles including spin, Pauli's exclusion principle and exchange in the electron-electron collision. Later Bethe elaborated and reviewed the subject ([Be32] and [Be33]). This chapter serves mainly to summarize some of the quantum mechanical results of Bethe and Møller for the interaction of charged particles with individual atoms. We use their equations as a basis for discussing the mode of energy deposition and calculations of ionization cross sections, excitation cross sections, energy spectra of fast and slow (resonance) electrons, and the average energy W per ionization of hydrogen. In the last section of this chapter we review briefly the interactions with complex atoms.

2. INCIDENT ELECTRONS COLLISION WITH FREE ELECTRONS

Møller ([Mø31], [Mø32a] and [Mø32b]) using Born's approximation developed a relativistic theory, including spin, for the interaction of electrons with free electrons. Møller's cross section equation for transfer of energy T to a free electron, Eq. 76, p. 569 of reference [Mø32a], may be written in the following form:

$$d\sigma = \frac{2\pi . e^4}{m_o v^2} \cdot \left[\frac{1}{T^2} + \frac{1}{(E-T)^2} - \frac{1}{T(E-T)} \cdot \frac{m_o c^2 (2E+m_o c^2)}{(E+m_o c^2)^2} + \frac{1}{(E+m_o c^2)^2} \right] \cdot dT \quad (1)$$

DANSK RESUMÈ.

I denne afhandling udvikles formuleringer af hurtige, ladede partiklers bremsning.

I kapitel I omtales de kvanteteoretiske resultater af vekselvirkningen mellem hurtige indfaldende elektroner og atomare elektroner. I kapitel II fremsaettes en matematisk formulering af den klassiske stødproces. Den meget naere overensstemmelse med de kvanteteoretiske resultater fremhaeves. I kapitel III bliver den teoretiske formulering, som blev udviklet i kapitel II, udvidet til at omfatte kompakte stoffer, der karakteriseres ved at alle oscillatorerne er af samme slags og indbyrdes koblede. Denne formulering afviger i mange henseender fra den, som først blev fremsat af Fermi. For eks. behandles energiabsorptionen og energitabet i form af Čerenkovstråling saerskilt. I kapitel IV udvides den teoretiske formulering fra kapitel II og kapitel III til at omfatte kompakte materialer, bestående af mange forskellige slags oscillatortyper. Den fremsatte behandling af stødproblemerne i kompakte materialer gør det muligt at behandle absorptionen og Čerenkovstrålingen saerskilt. Det viser sig da, at den hidtidige teori må korrigeres. For eksempel viser kvanteteoretiske betragtninger sig for kompakte stoffer at resultere i en relativistisk korrektion Δ_4, som varierer med energien af den indfaldende partikel. Denne meget vigtige korrektion Δ_4, som gaelder ganske almindeligt for alle stoffer, kan forklare den hidtidige uoverensstemmelse mellem bremseresultaterne for 10-30 MeV protoner og 100-1000 Mev protoner. Desuden resulterer betragtningerne i kapitel IV i en yderligere ny meget vigtig korrektion, Δ_3, som skyldes kobling mellem uensartede oscillatorer. De udledte formler bruges siden til at beregne bremsningen af protoner i helium og aluminium. Overensstemmelsen mellem de således fundne rent teoretiske vaerdier og de bedste eksperimenter er meget god.